Mathematical Models

Mathematics is used as a tool or language, enabling biological scientist to express their ideas so that quantitative prediction is possible, and these prediction are then compared with observational data.Mathematical models can contribute to both of the aims. As with many things in this good life, there are models and models. Dynamic deterministic models predict how a system unfolds with the passage of time-the time course of events; deterministic models make definite predictions (i.e. on 1st July the dry matter per unit area of the wheat crop will be 1 kg per sq. m.)without any associated probability distribution. Dynamic deterministic models comes in three types. We call these types teleonomic, empirical and mechanistic, although some would choose a different terminology. In terms of biology organizational hierarchy levels, teleonomic models look (mostly) upwards to the higher levels, empirical models examine a single level and mechanistic models look downwards, considering a level in relation to lower levels.Teleonomic models are sometimes called teleological or goal seeking. Empirical models belong to the category associated with curve fitting, regression and applying mathematical formulae directly to observational data, usually without being contained by scientific principles or any knowledge of mechanism. Mechanistic models are reductionist, concerned with mechanism and integrative; they contribute understanding and are sometimes called explanatory.

1. Mathematical models – objectives and contributions:

A mathematical model is an equation or a set of equation, whole solution provides a time space revolutions of the state variables i.e. physical behavior, in the framework of mathematical model of related feasible system. The equation derived by mathematical model can also called as state equation. The mathematical modelling essentially consists of translating real world problem into mathematical problems, solving mathematical problem and interpreting these solutions in language of real world.

A Mathematical model is an equation or set of equations which represents the behavior of a system. There is a correspondence between the variables of the model and observable quantities.Many laws for modeling work in biology in plants and crops sciences. We list some of the possible objectives and potential benefits associated with mathematical modeling. EM for Empirical model and MM for mechanistic model are used.

1. Hypothesis expressed in mathematical terms can provide a quantitative description and mechanistic understanding of a biological system. (MM)

2. A model required a completely define conceptual framework, and this may pinpoint areas where knowledge is lacking, and perhaps stimulate new ideas and experimental approaches. (MM)

3. A mathematical model especially if implemented in an easy –to –use computer program, may provide an excellent recipe by which recent research knowledge is made available to the farm manager or adviser(EM mostly ,some MM).

4. Agro-economic models may highlight the benefit of new crop management techniques suggested by recent research, thereby stimulating the adoption of more efficient production methods (MM, EM).

5. Modeling may lead to less ad hoc experimentation; models may make it possible to design experiments to answer particular questions, or to discriminate between alternative mechanisms (MM, EM).

6. In a system with several components, a model provides a means of bringing together knowledge about the parts, giving an integrated view of whole system behavior (MM).

7. Modeling can provide strategic and tactical support to a research programme , motivating scientists and encouraging collaboration (MM).

8. A model may provide a powerful means of summarizing data, and also a method for interpolation and cautions extrapolation (EM mostly, MM).

9. Observational data are becoming more precise, but also more complete use of such data (EM, MM).

10. The predictive power of a successful model can be used in many ways. For instance a model can be used to answer ‘what if . . . ? Questions what are the consequences on crop production for halving the maintenance requirements of plant tissue? What are the effects on crop yields of changing within plant transport resistance? However ,it should be remembered that the answer given by a model are , in a sense, built into it by hypothesis .Thus a model can be used to stimulate thought , but it may be dangerous to use a model to manage a research and development programme(MM).

Any given model is only likely to contribute under two or three of these ten points .However

2. Deterministic dynamic differential equation models:

It is assumed that the state of the system under investigation at time t is defined by q variables $X_1, X_2 \ldots\ldots, X_q$;these q variables are called state variables. The q state variable are independent; that is, it is not possible to derive one of the state variables,X say, from a knowledge of the values of the other state variables. The state variables represent properties or attributes of the system being considered (such as dry matter, number of cells, leaf area, starch content etc.). The choice of state variables is the **first** and most important assumption that the modeler makes. The scope of the model is defined by its state variables. The **next** step is to construct the q first-order ordinary differential equations that describe how the q state variables change with time t. These can be written formally as

$$\frac{dX_1}{dt} = f_1(X_1, X_2, \ldots, X_q; P; E)$$

$$\frac{dX_2}{dt} = f_2(X_1, X_2, \ldots, X_q; P; E)$$

$$\frac{dX_q}{dt} = f_q(X_1, X_2, \ldots, X_q; P; E).$$

The $f_1, f_2, \ldots, f_q$ denote functions of the state variables ,of a number of parameters which are indicated by P,and of environmental quantities denoted by E. the above equations are called ‘rate –state’ equations.

Denoting a single state variables by X, we can write its rate-state equations as,

$$\frac{dX}{dt} = inputs - outputs$$

The inputs are the terms that contribute positively to the rate of change of X and the outputs contribute negatively. Each terms of the right hand side of the above equation gives the rate of a process. In the plant science, all processes are either transport or chemical conversion.

Explicit time dependence

Suppose that one of the rate-state equations (1) takes the form

$$\frac{dX}{dt} = aX(b - t)$$

Where a and b are constants. Here the time variable t appears explicitly on the right hand side of the equation. The system is completely specified by the set of state variables and does not know what the time is, although one or more state variables may effectively be keeping track of time. However the elimination of one or more state variables may lead to reduced set of rate state equations with explicit time dependence, thus explicit time dependence can be viewed as representing hidden state variables. From above equation we could expand the set of state variables by defining a second state variable Y by

$$Y = b - t$$

From which t is easily eliminated by a single differentiation, giving

$$\frac{dY}{dt} = -1 \quad withY = batt = 0$$

The time independent rate stateequations are now

$$\frac{dX}{dt} = aXYand\frac{dY}{dt} = -1$$

$\frac{dX}{dt} = aX(b - t)$can be integrated to give as

$$X = X_0 exp\left[\frac{at(2b - t)}{2}\right]$$

$X=X_0$ at t=0. This function is known as the exponential quadratic and is sometimes employed in plant growth analysis.

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