Multiple Comparison Tests and Contrast Analysis (Ph.D. STAT 531)( Extension and Economics Students)

When the value of F in ANOVA table for any effect or interaction is found to be significant, it reveals that treatment effects or interaction differ significantly with respect to other effects .It is interesting to isolate the effects which differ significantly rather than others. Various tests have been sought for comparison of the difference arising from treatment means. These are known as the multiple comparison tests.

1. **Critical difference or least significant difference**
2. **Student-Newman Kuel test**
3. **Scheffe’s test**
4. **Duncan’s multiple range test**
5. **Tukey’s test**
6. **Dunnet’s t test**
7. **Critical difference**:

It is computed by the formula

CD= tα at error d.$f.\sqrt{}MSE(\frac{1}{n1}+\frac{1}{n2})$ in the case of unequal analysis of one way classification and CD= tα at error d.$f\sqrt{2MSE/n}$ in the case of two way classification if n1=n2, where n1 and n2 are the number of observations in I and II classes of one way analysis of variance. Two classes means differ significantly if their mean difference is larger than or equal to CD values, otherwise at par.

 Critical difference or least significant difference is considered to be appropriate for comparison of smaller number of treatments or class means. In fact, type I error increases as the number of treatments or class means increases. Type I error associated with the largest and smallest mean pair for five treatments is 27%, for ten treatments, it is 59% and for twenty treatments , it is 86%.

**Student-Newman Kuel test:**

This test is similar to critical difference except that for every pair of factor or treatment means,a separate value of q( K,error d.f.) is used instead of means from the largest difference to the the smallest in all possible pairs forming a triangular arrangement. Then prepare a list of the significant ranges as follows the largest mean and smallest mean pair.

Rk= qk (K,error d.f.)xSem

Where qk (K,error d.f.) means the value of percentage points of the studentized distribution at error d.f. and K means and Sem is the standard error of any treatment mean where Sem=$\sqrt{MSE/n}$ where n ,being the number of observations in each treatment.

Similarly, for the comparison of last but one largest mean and smallest mean, we have

Rk-1= qk (K-1,error d.f.) and in the last , we compute R2= qk (2,error d.f.) for comparison of mean of 2 Vs mean of 1.

**Scheffe’s test:**

In this test, we compute T=[(t-1)F{(t-1))n-t)}α].1/2.SED

Where t is the number of treatments,SED is the standard error of difference between two treatment means and F{(t-1))n-t)}α is the percentage points of F distribution at (t-1),(n-t) d.f.,being taken at α level of significance.

Two treatments will differ significantly if their difference is greater than or equal to T, otherwise not.

**Duncan’s multiple range test:**

It is used for comparison of all possible pairs of treatment means whether F is significant or not. It erquires a least significant value for comparison of means. First,arrange the treatment means in ascending order of magnitude,and then compute the standard error of treatment mean(SEm) as $\sqrt{MSE/n}$ where n is the number of replications. Now from the Statistical Table choose the studentized range (rp)for 2,3,4,… t treatments at error d.f..now, compute the short cut significant range (Rp) by the given formula:

Rp=rpxSem

Compute the difference between largest mean and the largest Rp. Declare all the means less than this value as significantly different from the largest mean. Then take the difference between second largest mean and the second largest Rp. Again declare all the means less than this value as significantly different from the second largest mean. If two treatment are remained use R2 value . For three treatments, use R3. Continue this process unless all the means are not tested for their significance. .In the last, line bars or alphabets are used to identify the significant means.

**Tukey’s test:**

 In this test ,we compute a difference D= QxSEm which is similar to Newman-Kuel test.The value of Q is taken as percentage point of studentized distribution at error d.f. and α level of significance. SEm is the standard error of any treatment mean. Then we again compute the pairwise treatment mean differences. If the mean difference is found to be larger tah the value of D, the declare two mean difference is significabnt otherwise not.

**Dunnet’s t test:**

This test is valid only when we have one control and several other treatments to be compared .This test is applicable in medicine or industrial experiments . It is on the similar basis of t test.

**Factorial Experiments**

Factorial experiments involve simultaneously more than one factor and each factor is at two or more levels. Several factors affect simultaneously the characteristic under study in factorial experiments and the experimenter is interested in the main effects and the interaction effects among different factors. First, we consider an example to understand the utility of factorial experiments. Example: Suppose the yield from different plots in an agricultural experiment depends upon (i) variety of crop and (ii) type of fertilizer. Both the factors are in the control of the experimenter. (iii) Soil fertility. This factor is not in the control of the experimenter. In order to compare different crop varieties - assign it to different plots keeping other factors like irrigation, fertilizer, etc. fixed and the same for all the plots. - The conclusions for this will be valid only for the crops grown under similar conditions with respect to the factors like fertilizer, irrigation etc. In order to compare different fertilizers (or different dosage of fertilizers) - sow single crop on all the plots and vary the quantity of fertilizer from plot to plot. - The conclusions will become invalid if different varieties of the crop are sown. - It is quite possible that one variety may respond differently than another to a particular type of fertilizer. Suppose we wish to compare - two crop varieties – a and b, keeping the fertilizer fixed and - three varieties of fertilizers – A, B and C. This can be accomplished with two randomized block designs ( RBD ) by assigning the treatments at random to three plots in any block and two crop varieties at random.

If the number of levels for each factor is the same, we call it is a symmetrical factorial experiment. If the number of levels of each factor is not the same, then we call it as asymmetrical or mixed factorial experiment.

We consider only symmetrical factorial experiments.

Through the factorial experiments, we can study - the individual effect of each factor and - interaction effect.

Now we consider a 22 factorial experiment with an example and try to develop and understand the theory and notations through this example.

General notation for representing the factors is to use capital letters, e.g., A, B, C etc. and levels of a factor are represented in small letters. For example, if there are two levels of A, they are denoted as a 0 and a 1 . Similarly, the two levels of B are represented as b0 and b1 . An important point to remember is that the factorial experiments are conducted in the design of an experiment. For example, the factorial experiment is conducted as an CRD, RBD.or LSD.

Factorial experiments with factors at two levels ( 2 2 factorial experiment):

Suppose in an experiment, the different varieties (V0 and V1) and different doses of fertilizers ( F0 and F1) is to be laid out, each factor consists of two levels. Then it is called 2X2 factorial experiment. This experiment has four treatment combinations which are as V0F0, V1F0,V0F1 and V1F1. The differential response of the Factor V in the presence of F0 which is equal to V1F0-V0F0. Similarly The differential response of the Factor V in the presence of F1 which is equal to V1F1-V0F1. These two effects are called simple effects. The mean of these two simple effects is called main effect of the factor V. In the same way we find the main effect of the factor F. The half of the difference between two simple effects from II to I will provide the interaction effects of the factor VxF. Thus, the main effect and interaction effect along with ANOVA of the two factors V and F are given below:

 Main effect of the factor V= $\frac{1}{2}$[(V-1)(F+1)

 Main effect of the factor F= $\frac{1}{2}$[(V+1)(F-1)

 Interaction effect of the factor VXF= $\frac{1}{2}$[(V-1)(F-1)

 These effects can also be shown with the help of a following table:

|  |  |  |
| --- | --- | --- |
| Effects |  V0F0 V1F0 V0F1 V1F1 |  Divisor |
| Mean |  + + + + |  4 |
| V | * + - +
 |  2 |
| F | * - + +
 |  2 |
| VXF |  + - - + |  2 |

 If this experiment is conducted in RCBD, then the ANOVA table can be displayed as Follows:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  SV | d.f | S.S | M.S | Fcal | Ftab |
|  Replication | r-1 | SSR | MSR |  |  |
| V | v-1 | SSV | MSV | MSV/MSE |  |
| F | f-1 | SSF | MSF | MSF/MSE |  |
| VXF | (v-1)(f-1) | SS(VF) | MS(VF) | MS(VF)/MSE |  |
| Error |  subtraction | SSE | MSE |  |  |
| Total  | rvf-1 |  |  |  |  |

If calculated value of any effect is found to be larger than its tabulated value at 5% or 1% level of significance, the null hypothesis is rejected and thus, we conclude that the effects are found to be significant. The crical difference will be used to identify the significant and at par treatments as well as interactions.

 This kind of exercise can be demonstrated for the number of increased factors and their increased number of levels.

Contrasts and Analysis of Variance

The main technique adopted for the analysis and interpretation of the data collected from an experiment is the analysis of variance technique that essentially consists of partitioning the total variation in an experiment into components ascribable to different sources of variation due to the controlled factors and error. Analysis of variance clearly indicates a difference among the treatment means. The objective of an experiment is often much more specific than merely determining whether or not all of the treatments give rise to similar responses. For examples, a chemical experiment might be run primarily to determine whether or not the yield of the chemical process increases as the amount of the catalyst is increased. A medical experimenter might be concerned with the efficacy of each of several new drugs as compared to a standard drug. A nutrition experiment may be run to compare high fiber diets with low fiber diets. A plant breeder may be interested in comparing exotic collections with indigenous cultivars. An agronomist may be interested in comparing the effects of bio fertilizers and chemical fertilizers. An water technologist may be interested in studying the effect of nitrogen with Farm Yard Manure over the nitrogen levels without farm yard manure in presence of irrigation.

Contrast:

The linear combination of the treatment means or total whose sum of the coefficients is equal to zero. For instance T1-2T2+T 3 is a contrast.

 Orthogonal contrast:

Two contrast C1 and C2 are said to orthogonal if the sum of their multiplication of the treatment means or totals, coefficients is equal to zero. For example A and B are two orthogonal contrast if A=1/2[ (a-1)(b+1) and B=1/2[(a=1)(b-1).

 Mutually orthogonal contrast:

 If in an experiment, there are more than two orthogonal contrast exist, it is called mutually orthogonal contrast. For example in the factorial experiment of 22,all V,F and VXF are mutually orthogonal contrast.

The analysis of variance table for two factors A with a levels and B with b levels with r

Replications tried in RBD will be as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of Variation | d.f | SS | MS | Fcal |
| Replication | r-1 | SSR | MSR |  |
| Factor A | a-1 | SSA | MSA | MSA/MSE |
| Factor B | b-1 | SSB | MSB | MSB/MSE |
| Interaction AB | (a-1)(b-1) | SS(AB) | MS(AB) | MS(AB)/MSE |
| Error | (r-1)(ab-1) | SS(Error) | MSE |  |
| Total | rab-1 | TSS Corrected |  |  |

As in the previous designs calculate the replication totals to calculate the SSR, TSS in the usual way. To calculate SSA, SSB and SS(AB), form a two way table A X B by taking the levels of A in rows and levels of B in the columns. To get the values in this table the missing factor is replication. That is by adding over replication we can form this table.

GT= Grand total of all observations

CF= (GT)2/rab

SSR=ΣRi2/ab-CF

Two way table of AXB

|  |  |  |  |
| --- | --- | --- | --- |
| A/B | b0 | b1 | Total |
| a0 | a0bo | a0b1 | Ao |
| a1 | a1b0 | a1b1 | A1 |
| Total | B0 | B1 | GT |

SSA= (A0+A1)2/bxr-CF

SSB= (B0+B1)2/axr-CF

SS(AB)=(aob0)2+(a0b1)2+(a1b0)2+(a1b1)2/r-CF-SSA-SSB

SS(Error)= By subtraction all the Sum of Squares from total SS Corrected.

23 Factorial Experiment in RBD

23 factorial experiment mean three factors each at two levels. Suppose the three factors

are A, B and C are tried with two levels the total number of combinations will be eight i.e. a0b0c0, a0b0c1, a0b1c0, a0b1c1, a1b0c0, a1b0c1, a1b1c0 and a1b1c1.

The allotment of these eight treatment combinations will be as allotted in RBD. That is

each block is divided into eight experimental units. By using the random numbers these eight combinations are allotted at random for each block separately.

The analysis of variance table for three factors A with a levels, B with b levels and C with c levels with r replications tried in RBD will be as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of Variation | d.f | SS | MS | Fcal |
| Replication | r-1 | SSR | MSR |  |
| Factor A | a-1 | SSA | MSA | MSA/MSE |
| Factor B | b-1 | SSB | MSB | MSB/MSE |
| Factor C | (a-1)(b-1) | SSC | MSC | MSC/MSE |
| Interaction AB | (a-1)(b-1) | SS(AB) | MS(AB) | MS(AB)/MSE |
| Interaction AC | (a-1)(c-1) | SS(AC) | MS(AC) | MS(AC)/MSE |
| Interaction BC | (b-1)(c-1) | SS(BC) | MS(BC) | MS(BC)/MSE |
| Interaction ABC | (a-1)(b-1)(c-1) | SS(ABC) | MS(ABC) | MS(ABC)/MSE |
| Error | (r-1)(abc-1) | SS(Error) | MSE |  |
| Total | rabc-1 | TSS Corrected |  |  |

As in the previous designs calculate the replication totals to calculate the CF, RSS, TSS, overall TrSS in the usual way. To calculate ASS, BSS, CSS, ABSS, ACSS, BCSS and ABCSS, form three two way tables A X B, AXC and BXC.

AXB two way table can be formed by taking the levels of A in rows and levels of B in

the columns. To get the values in this table the missing factor is replication. That is by adding over replication we can form the table.

AXB Two way Table:

|  |  |  |  |
| --- | --- | --- | --- |
| A/B | b0 | b1 | Total |
| a0 | a0bo | a0b1 | Ao |
| a1 | a1b0 | a1b1 | A1 |
| Total | B0 | B1 | GT |

AXC Two way table:

|  |  |  |  |
| --- | --- | --- | --- |
| A/C | c0 | c1 | Total |
| a0 | a0co | a0c1 | Ao |
| a1 | a1c0 | a1c1 | A1 |
| Total | C0 | C1 | GT |

By making two way table of other interactions, we can determine the sum of squares of each effect and consequently the mean squares,and then finally error mean squares.With the help of ANOVA table given above, we can test the treatment effects and ultimately each main effect and its interactions to other factors.