Factorial experiments – factor and levels – types – symmetrical and

asymmetrical – simple, main and interaction effects – advantages and disadvantages

Factorial Experiments

When two or more number of factors are investigated simultaneously in a single

experiment such experiments are called as factorial experiments.

Terminologies

1. Factor: Factor refers to a set of related treatments. We may apply of different

doses of nitrogen to a crop. Hence nitrogen irrespective of doses is a factor.

2. Levels of a factor: Different states or components making up a factor are known

as the levels of that factor. eg different doses of nitrogen.

Types of factorial Experiment

A factorial experiment is named based on the number of factors and levels of factors. For example, when there are 3 factors each at 2 levels the experiment is known as 2 X

2 X 2 or 23 factorial experiments. If there are 2 factors each at 3 levels then it is known as 3 X 3 or 32 factorial experiment.

In general if there are n factors each with p levels then it is known as pn

factorial experiment.

• For varying number of levels the arrangement is described by the product. For

example, an experiment with 3 factors each at 2 levels, 3 levels and 4 levels

respectively then it is known as 2 X 3 X 4 factorial experiment.

• If all the factors have the same number of levels the experiment is known as

symmetrical factorial otherwise it is called as mixed factorial.

• Factors are represented by capital letters. Treatment combinations are usually

by small letters.

• For example, if there are 2 varieties v0 and v1 and 2 dates of sowing d0 and

d1 the treatment combinations will be

• vodo, v1do, v1do and v1d1.

Simple and Main Effects

Simple effect of a factor is the difference between its responses for a fixed level of

other factors. Main effect is defined as the average of the simple effects. Interaction is defined as the dependence of factors in their responses. Interaction is measured as the mean of the differences between simple effects.

Advantages

1. In such type of experiments we study the individual effects of each factor and

their interactions.

2. In factorial experiments a wide range of factor combinations are used.

3. Factorial approach will result in considerable saving of the experimental

resources, experimental material and time.

Disadvantages

1. When number of factors or levels of factors or both are increased, the number of

treatment combinations increases. Consequently block size increases. If block size

increases it may be difficult to maintain homogeneity of experimental material.

This will lead to increase in experimental error and loss of precision in the

experiment.

2. All treatment combinations are to be included for the experiment irrespective of

its importance and hence this results in wastage of experimental material and

time.

3. When many treatment combinations are included the execution of the experiment

and statistical analysis become difficult.

22 Factorial Experiments in RBD

22 factorial experiment means two factors each at two levels. Suppose the two factors are A and B and both are tried with two levels the total number of treatment combinations will be four i.e. a0b0, a0b1, a1b0 and a1b1.

The allotment of these four treatment combinations will be as allotted in RBD. That is

each block is divided into four experimental units. By using the random numbers these four combinations are allotted at random for each block separately.

The analysis of variance table for two factors A with a levels and B with b levels with r

replications tried in RBD will be as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of Variation | d.f | SS | MS | Fcal |
| Replication | r-1 | SSR | MSR |  |
| Factor A | a-1 | SSA | MSA | MSA/MSE |
| Factor B | b-1 | SSB | MSB | MSB/MSE |
| Interaction AB | (a-1)(b-1) | SS(AB) | MS(AB) | MS(AB)/MSE |
| Error | (r-1)(ab-1) | SS(Error) | MSE |  |
| Total | rab-1 | TSS Corrected |  |  |

As in the previous designs calculate the replication totals to calculate the SSR, TSS in the usual way. To calculate SSA, SSB and SS(AB), form a two way table A X B by taking the levels of A in rows and levels of B in the columns. To get the values in this table the missing factor is replication. That is by adding over replication we can form this table.

GT= Grand total of all observations

CF= (GT)2/rab

SSR=ΣRi2/ab-CF

Two way table of AXB

|  |  |  |  |
| --- | --- | --- | --- |
| A/B | b0 | b1 | Total |
| a0 | a0bo | a0b1 | Ao |
| a1 | a1b0 | a1b1 | A1 |
| Total | B0 | B1 | GT |

SSA= (A0+A1)2/bxr-CF

SSB= (B0+B1)2/axr-CF

SS(AB)=(aob0)2+(a0b1)2+(a1b0)2+(a1b1)2/r-CF-SSA-SSB

SS(Error)= By subtraction all the Sum of Squares from total SS Corrected.

23 Factorial Experiment in RBD

23 factorial experiment mean three factors each at two levels. Suppose the three factors

are A, B and C are tried with two levels the total number of combinations will be eight i.e. a0b0c0, a0b0c1, a0b1c0, a0b1c1, a1b0c0, a1b0c1, a1b1c0 and a1b1c1.

The allotment of these eight treatment combinations will be as allotted in RBD. That is

each block is divided into eight experimental units. By using the random numbers these eight combinations are allotted at random for each block separately.

The analysis of variance table for three factors A with a levels, B with b levels and C with c levels with r replications tried in RBD will be as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of Variation | d.f | SS | MS | Fcal |
| Replication | r-1 | SSR | MSR |  |
| Factor A | a-1 | SSA | MSA | MSA/MSE |
| Factor B | b-1 | SSB | MSB | MSB/MSE |
| Factor C | (a-1)(b-1) | SSC | MSC | MSC/MSE |
| Interaction AB | (a-1)(b-1) | SS(AB) | MS(AB) | MS(AB)/MSE |
| Interaction AC | (a-1)(c-1) | SS(AC) | MS(AC) | MS(AC)/MSE |
| Interaction BC | (b-1)(c-1) | SS(BC) | MS(BC) | MS(BC)/MSE |
| Interaction ABC | (a-1)(b-1)(c-1) | SS(ABC) | MS(ABC) | MS(ABC)/MSE |
| Error | (r-1)(abc-1) | SS(Error) | MSE |  |
| Total | rabc-1 | TSS Corrected |  |  |

As in the previous designs calculate the replication totals to calculate the CF, RSS, TSS, overall TrSS in the usual way. To calculate ASS, BSS, CSS, ABSS, ACSS, BCSS and ABCSS, form three two way tables A X B, AXC and BXC.

AXB two way table can be formed by taking the levels of A in rows and levels of B in

the columns. To get the values in this table the missing factor is replication. That is by adding over replication we can form the table.

AXB Two way Table:

|  |  |  |  |
| --- | --- | --- | --- |
| A/B | b0 | b1 | Total |
| a0 | a0bo | a0b1 | Ao |
| a1 | a1b0 | a1b1 | A1 |
| Total | B0 | B1 | GT |

AXC Two way table:

|  |  |  |  |
| --- | --- | --- | --- |
| A/C | c0 | c1 | Total |
| a0 | a0co | a0c1 | Ao |
| a1 | a1c0 | a1c1 | A1 |
| Total | C0 | C1 | GT |

By making two way table of other interactions, we can determine the sum of squares of each effect and consequently the mean squares,and then finally error mean squares.With the help of ANOVA table given above, we can test the treatment effects and ultimately each main effect and its interactions to other factors.

Split-plot Design

In field experiments certain factors may require larger plots than for others. For

example, experiments on irrigation, tillage, etc requires larger areas. On the other hand

experiments on fertilizers, etc may not require larger areas. To accommodate factors

which require different sizes of experimental plots in the same experiment, split plot

design has been evolved.

In this design, larger plots are taken for the factor which requires larger plots. Next each of the larger plots is split into smaller plots to accommodate the other factor. The different treatments are allotted at random to their respective plots. Such arrangement is called split plot design. In split plot design the larger plots are called main plots and smaller plots within the larger plots are called as sub plots. The factor levels allotted to the main plots are main plot treatments and the factor levels allotted to sub plots are called as sub plot treatments

Layout and analysis of variance table

First the main plot treatment and sub plot treatment are usually decided based on

the needed precision. The factor for which greater precision is required is assigned to the sub plots. The replication is then divided into number of main plots equivalent to main plot treatments. Each main plot is divided into subplots depending on the number of sub plot treatments. The main plot treatments are allocated at random to the main plots as in the case of RBD. Within each main plot the sub plot treatments are allocated at random as in the case of RBD. Thus randomization is done in two stages. The same procedure is followed for all the replications independently.

The analysis of variance will have two parts, which correspond to the main plotsand sub-plots. For the main plot analysis, replication X main plot treatments table is formed. From this two-way table sum of squares for replication, main plot treatments and error (a) are computed. For the analysis of sub-plot treatments, main plot X sub-plot treatments table is formed. From this table the sums of squares for sub-plot treatments and interaction between main plot and sub-plot treatments are computed. Error (b) sum of squares is found out by residual method. The analysis of variance table for a split plot design with m main plot treatments and s sub-plot treatments is given below.

Analysis of variance for split plot with factor A with m levels in main plots and factor B with s levels in sub-plots will be as follows:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Source of Variation | d.f | SS | MS | Fcal |
| Replication | r-1 | SSR | MSR |  |
|  A | a-1 | SSA | MSA | MSA/MSE (I) |
| Error I | (r-1)(a-1) | SS(Error I) |  |  |
|  B | b-1 | SSB | MSB | MSB/MSE(II) |
| Interaction AB | (a-1)(b-1) | SS(AB) | MS(AB) | MS(AB)/MSE(II) |
| Error II | a(r-1)(b-1) | SS(Error II) | MSE |  |
| Total | rab-1 | TSS Corrected |  |  |

 Like factorial experiments, we will form two basic tables (i) Replications Vs Main plot and (ii) Main plot VS sub plot treatments. Keeping all the sum of squares in ANOVA table given earlier , we can test each main effect with the help of Error I and Sub plot treatment effets and its interaction with main plot treatmentby error II and consequently draw the conclusions about the given data.